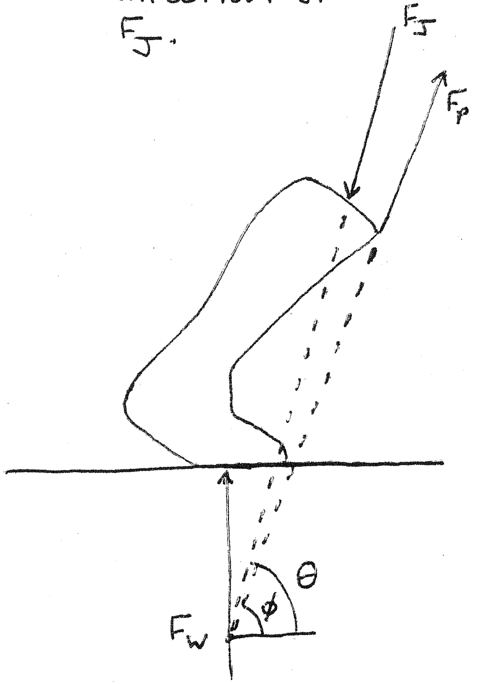
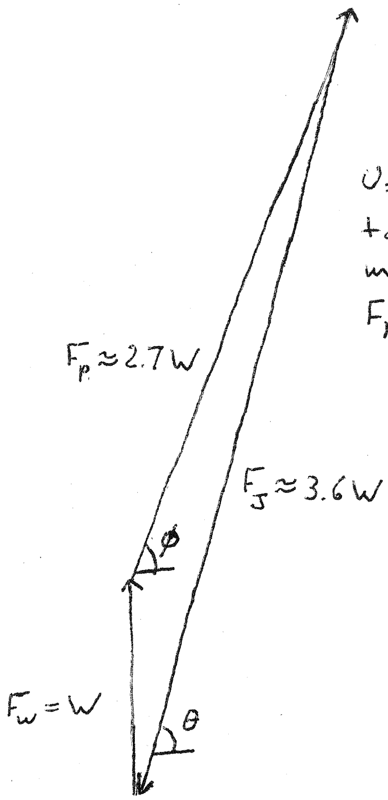
2.1

Find point of concurrency to determine the direction of F_J.



Use a vector diagram to estimate the magnitude of F_J and F_p.



2.2

$$\sum M_H = I_H \alpha$$

$$(I_{HAT})_{Hip}\alpha = T_H - d_{HAT} m_{HAT} g$$

$$T_H = (I_{HAT})_{Hip}\alpha + d_{HAT} m_{HAT} g$$

$$(I_{HAT})_{Hip} = (k_{HAT})^2_{Hip} m_{HAT}$$

$$(k_{HAT})_{Hip} = .621 \times .52 \times 1.8 = 0.581\ m\ , \quad m_{HAT} = .678 \times 70 = 47.46\ kg$$

$$d_{HAT} = .374 \times .52 \times 1.8 = .350\ m$$

$$(I_{HAT})_{Hip} = .581^2 \times 47.46 = 16.02\ kg \cdot m^2$$

$$T_H = 16.02 \times 3 + .350 \times 47.46 \times 9.81 = 48.1 + 162.9 = 211.0\ N \cdot m$$

2-3

About the hip

$$\sum M_H = I_H \alpha$$

$$T_H - m_T g d_T - m_{LL} g (d_{LL} + l_T) = (I_T + I_{LL})_H \alpha$$

$$(I_{LL})_H = (I_{LL})_{cm} + (l_T + d_{LL})^2 m_{LL}$$

$$T_H = ((I_T)_H + (I_{LL})_{cm} + (l_T + d_{LL})^2 m_{LL})\alpha + m_T g d_T + m_{LL} g (d_{LL} + l_T)$$

$$T_H = (.4 + .05 + 5(.40 + .20)^2)(2) + (8)(9.81)(.18) + (5)(9.81)(.20 + .40)$$

$$T_H = 48\ N \cdot m$$

For the lower leg

$$\sum F_y = m_{LL}(a_{LL})_y$$

$$F_{Ky} - m_{LL} a_G = m_{LL}(l_T + d_{LL})\alpha$$

$$F_{Ky} = m_{LL}(l_T + d_{LL})\alpha + m_{LL} a_G$$

$$F_{Ky} = 5(.4 + .2)(2) + 5(9.81)$$

$$F_{Ky} = 55\ N$$

About the mass center of the lower leg

$$\sum M_{GL} = I_{GL}\alpha = T_K - d_{LL} F_{Ky}$$

$$T_K = I_{GL}\alpha + d_{LL} F_{Ky}$$

$$T_K = 5(2) + .2(55)$$

$$T_K = 11\ N \cdot m$$

About the hip

$$\sum M_H = I_H \alpha$$

$$T_H + m_T d_T (a_T - a_G) + m_{LL}(l_T + d_{LL})(a_{LL} - a_G) = (I_T + I_{LL})_H \alpha$$

$$a_T = \alpha d_T \qquad a_{LL} = a_K = \alpha l_T$$

$$(I_{LL})_H = (I_{LL})_{cm} + m_{LL}(l_T + d_{LL})^2$$

$$T_H = [I_T + (I_{LL})_{cm} + m_{LL}(l_T + d_{LL})^2]\alpha - m_T d_T(\alpha d_T - a_G) - m_{LL}(l_T + d_{LL})(\alpha l_T - a_G)$$

$$T_H = [.4 + .05 + 5(.4 + .2)^2](3) - (8)(.18)[(3)(.18) - 9.81] - 5(.4 + .2)[(3)(.4) - 9.81]$$

$$T_H = 46\ N \cdot m$$

For the lower leg

$$\sum F_y = m_{LL}(a_{LL})_y \qquad a_K = \alpha l_T$$

$$F_{Ky} = m_{LL}\alpha l_T + m_{LL} a_G = m_{LL}(\alpha l_T + a_G)$$

$$F_{Ky} = 5[3(.4) + 9.81]$$

$$F_{Ky} = 55\ N$$

About the mass center of the lower leg

$$\sum M_{GL} = I_{GL}\alpha = T_K - d_{LL} F_{Ky}$$

$$T_K = I_{GL}\alpha + d_{LL} F_{Ky}$$

$$T_K = .05(3) + .2(55)$$

$$T_K = 11\ N \cdot m$$

2.5

moments about the shoulder

$$M_S - m_L g d_L - (m_h + m_b) g (L_L + d_h) = \sum I_{cm}\alpha + \left|\vec{d} \times m\vec{r}\right|$$

RHS of the equation

$$(I_u + I_L)\alpha + d_u m_u a_u + m_L a_L \sqrt{L_u^2 + d_L^2} + (m_h + m_b) a_h \sqrt{L_u^2 + (L_L + d_h)^2}$$

$$a_u = d_u \alpha \quad a_L = \alpha\sqrt{L_u^2 + d_L^2} \quad a_h = \alpha\sqrt{L_u^2 + (L_L + d_h)^2}$$

$$(I_u + I_L)\alpha + d_u^2 m_u \alpha + (L_u^2 + d_L^2) m_L \alpha + \left(L_u^2 + (L_L + d_h)^2\right)(m_h + m_b)\alpha$$

solving for the moment

$$M_S = m_L g d_L + (m_h + m_b) g (L_L + d_h) + \alpha\left[I_u + I_L + d_u^2 m_u + (L_u^2 + d_L^2) m_L + \left(L_u^2 + (L_L + d_h)^2\right)(m_h + m_b)\right]$$

substituting values yields

$$M_S = 12.02\ N \cdot m$$

moments about the elbow

$$M_E - m_L g d_L - (m_h + m_b) g (L_L + d_h) = \sum I_{cm}\alpha + \left|\vec{d} \times m\vec{r}\right|$$

$$M_E - m_L g d_L - (m_h + m_b) g (L_L + d_h) = d_L^2 m_L \alpha + \left(L_L + d_h\right)^2 (m_h + m_b)\alpha$$

$$M_E = m_L g d_L + (m_h + m_b) g (L_L + d_h) + d_L^2 m_L \alpha + \left(L_L + d_h\right)^2 (m_h + m_b)\alpha$$

substituting values yields

$$M_E = 10.66\, N \cdot m$$

$$T_h = [(I_T)_H + (I_{LL})_H]\alpha$$

$$(I_T)_H = (I_T)_{GT} + m_T {d_T}^2 = 4.2 + .045(8)^2 = 7.08\, lb \cdot sec^2 \cdot in$$

$$(I_{LL})_H = (I_{LL})_{GL} + m_{LL}(l_T + d_{LL})^2 = 1.4 + .025(18+8)^2 = 18.3\, lb \cdot sec^2 \cdot in$$

$$T_H = (7.08 + 18.3) \times 20 = 507.6\, in \cdot lb$$

$$\sum F_x = m_{LL}\ddot{x}_{ll} = F_{Kx}; \quad \sum F_y = m_{LL}\ddot{y}_{ll} = F_{Ky} - m_{LL}g$$

$$\ddot{x}_{ll} = (l_T + d_{LL})\alpha = (18+8) \times 20 = 520 \frac{in}{sec^2}; \quad \ddot{y}_{ll} = 0$$

$$F_{Kx} = m_{LL}\ddot{x}_{ll}\ (1) = .025 \times 520 = 13\, lbf$$

$$\sum M_{GL} = (I_{LL})_{GL}\ddot{\theta}_{LL} = T_K - a_{LL}F_{Kx}$$

$$T_K = (I_{LL})_{GL}\ddot{\theta}_{LL} + d_{LL}F_{Kx} = 1.4 \times 20 + 8 \times 13 = 132\, in \cdot lb$$

2.7

$$T_H = (I_T)_{GT}\alpha + (d_T)m_T a_{GTx} + (l_T + d_{LL})m_{LL}a_{GLx}$$

$$T_H = (I_T)_{GT}\alpha + (d_T)^2 m_T\alpha + (l_T + d_{LL})(l_T)m_{LL}\alpha$$

$$T_H = [(I_T)_{GT} + (d_T)^2 m_T + (l_T + d_{LL})(l_T)m_{LL}]\alpha$$

$$T_H = [4.2 + (8)^2(.045) + (18+8)(18)(.025)]20 = 376\ in \cdot lb$$

For the lower leg

$$\sum F_y = m_{LL}(a_{LL})_y \qquad a_K = \alpha l_T$$

$$F_{Ky} = m_{LL}\alpha l_T + m_{LL}a_G = m_{LL}(\alpha l_T + a_G)$$

$$F_{Ky} = .025[20(18) + 9.81]$$

$$F_{Ky} = 9.25\ lbf$$

About the mass center of the lower leg

$$\sum M_{GL} = I_{GL}\alpha = T_K - d_{LL}F_{Ky}$$

$$T_K = I_{GL}\alpha + d_{LL}F_{Ky}$$

$$T_K = 1.4(20) + 8(9.25)$$

$$T_K = 102\ lb \cdot in$$

2-8

By forward difference approximation

$$v_{x1}(t) = \frac{x_1(t+\Delta t) - x_1(t)}{\Delta t}$$

Apply the above to calculate v_{x2}, v_{y1}, v_{y2}

$$\|\vec{\omega}\| = \frac{\|\vec{v}\|}{\|\vec{r}\|}$$

$$\omega(t) = \frac{\sqrt{(v_{x2}(t) - v_{x1}(t))^2 + (v_{y2}(t) - v_{y1}(t))^2}}{\sqrt{(x_2(t) - x_1(t))^2 + (y_2(t) - y_1(t))^2}}$$

$$\alpha(t) = \frac{\omega(t+\Delta t) - \omega(t)}{\Delta t}$$

t, s	x_1, cm	y_1, cm	x_2, cm	y_2, cm	v_{x1}, cm/s	v_{y1}, cm/s	v_{x2}, cm/s	v_{y2}, cm/s	ω, rad/s	α, rad/s²
0.0	131.8	6.1	162.6	6.9	8.0	1.0	-23.0	68.0	**2.396**	**1.987**
0.1	132.6	6.2	160.3	13.7	-2.0	-3.0	-21.0	69.0	**2.595**	**3.110**
0.2	132.4	5.9	158.2	20.6	-6.0	-7.0	-52.0	66.0	**2.906**	**-8.699**
0.3	131.8	5.2	153.0	27.2	-4.0	4.0	-54.0	41.0	**2.036**	
0.4	131.4	5.6	147.6	31.3						

2-9

By forward difference approximation

$$v_{x1}(t) = \frac{x_1(t+\Delta t) - x_1(t)}{\Delta t}$$

Apply the above to calculate the other velocities

$$\|\vec{\omega}\| = \frac{\|\vec{v}\|}{\|\vec{r}\|} \qquad \omega(t) = \frac{\sqrt{(v_{x2}(t) - v_{x1}(t))^2 + (v_{y2}(t) - v_{y1}(t))^2}}{\sqrt{(x_2(t) - x_1(t))^2 + (y_2(t) - y_1(t))^2}}$$

$$\omega_{S/T} = \omega_T - \omega_S$$

$$\vec{\omega}_{S/T} = \frac{\|\vec{v}_T\|}{\|\vec{r}_T\|} - \frac{\|\vec{v}_S\|}{\|\vec{r}_S\|}$$

thigh

t, sec	**x1, cm**	**y1, cm**	**x2, cm**	**y2, cm**	**vx1, cm/s**	**vy1, cm/s**	**vx2, cm/s**	**vy2, cm/s**	**ω, rad/s**
0.05	51.0	85.1	57.0	57.9	52.0	0.0	66.0	-40.0	1.52
0.10	53.6	85.1	60.3	55.9	54.0	2.0	70.0	4.0	0.54
0.15	56.3	85.2	63.8	56.1	52.0	0.0	68.0	6.0	0.57
0.20	58.9	85.2	67.2	56.4					

shank

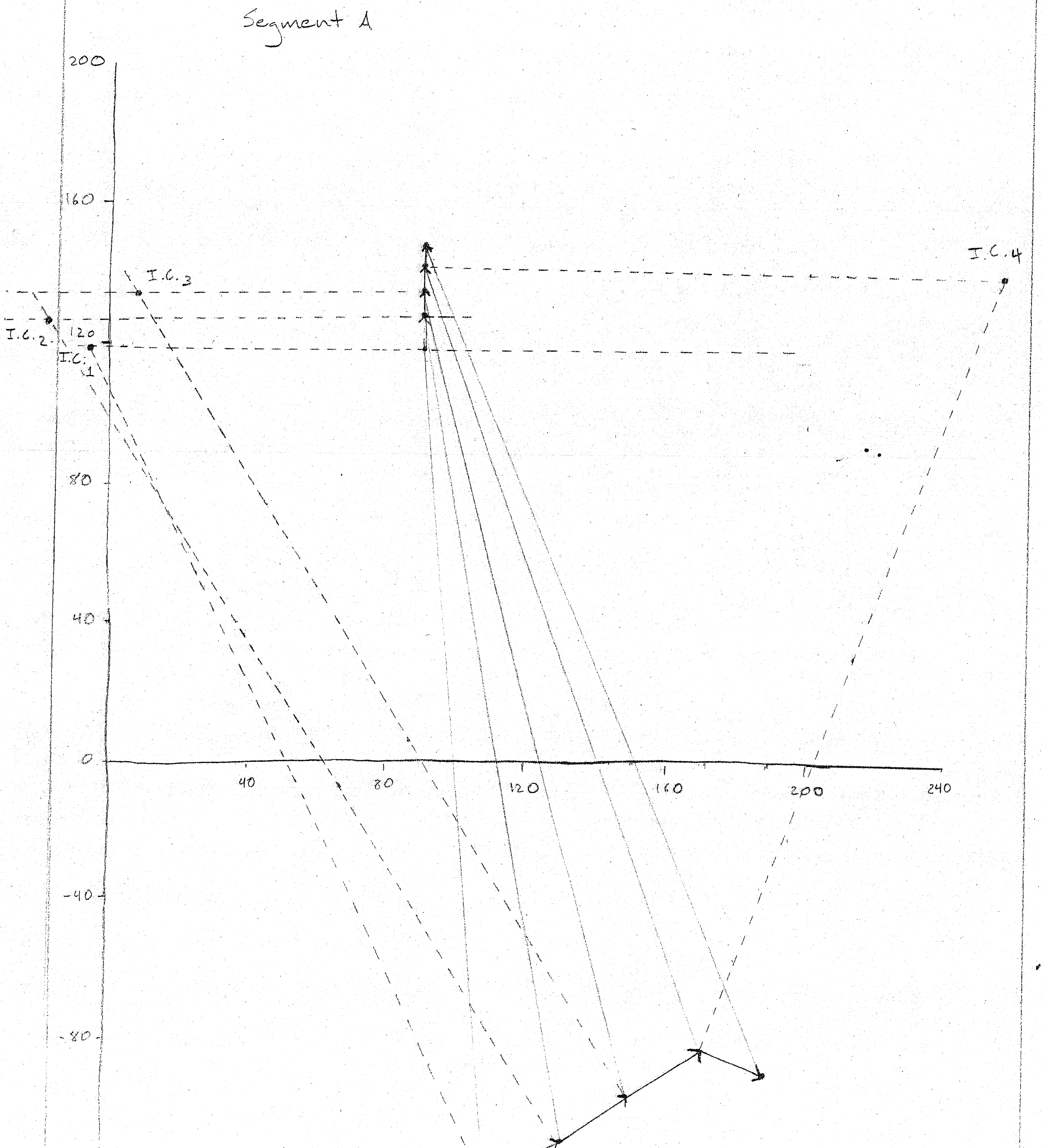
t, sec	**x3, cm**	**y3, cm**	**x4, cm**	**y4, cm**	**vx3, cm/s**	**vy3, cm/s**	**vx4, cm/s**	**vy4, cm/s**	**ω, rad/s**
0.05	57.9	45.8	57.9	10.8	72.0	4.0	80.0	4.0	0.23
0.10	61.5	46.0	61.9	11.0	72.0	6.0	78.0	6.0	0.17
0.15	65.1	46.3	65.8	11.3	12.0	6.0	78.0	6.0	1.89
0.20	65.7	46.6	69.7	11.6					

shank with respect to the thigh

t, sec	**ω, rad/s**
0.05	1.29
0.10	**0.37**
0.15	**-1.32**
0.20	

2.10

Calculate the locations of the average centers of rotation in a coordinate system defined on segment A.



Segment A

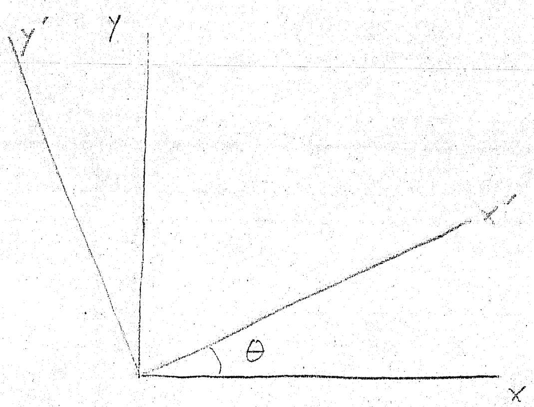
	I.C. (global) x	I.C. (global) y	Segment A Origin (global) x	Segment A Origin (global) y
1	-5.6	118.4	110.5	-119.5
2	-17.9	126.2	131.8	-108.2
3	8.2	134.2	151.1	-95.9
4	257.8	140.0	171.0	-82.7

I.C. w.r.t. Segment A Origin (global)

	x	y
1	-116.1	237.9
2	-149.7	234.4
3	-142.9	230.1
4	86.8	222.7

} translated coordinates

Rotation



$$\{\underline{X}\}' = [R]\{\underline{X}\}$$

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

	Theta (degrees)
1	5.02
2	10.22
3	15.02
4	19.99

I.C. w.r.t. Segment A Coordinate System

	x	y
	-94.83	247.15
	-105.76	257.23
	-78.38	259.27
	157.71	179.61
Average	-30.32	235.82

This process is then repeated for segment B.

I.C. w.r.t. Segment A Coordinate System

x	y
-58.3	256.1
3.31	112
28.0	61.4

2-11

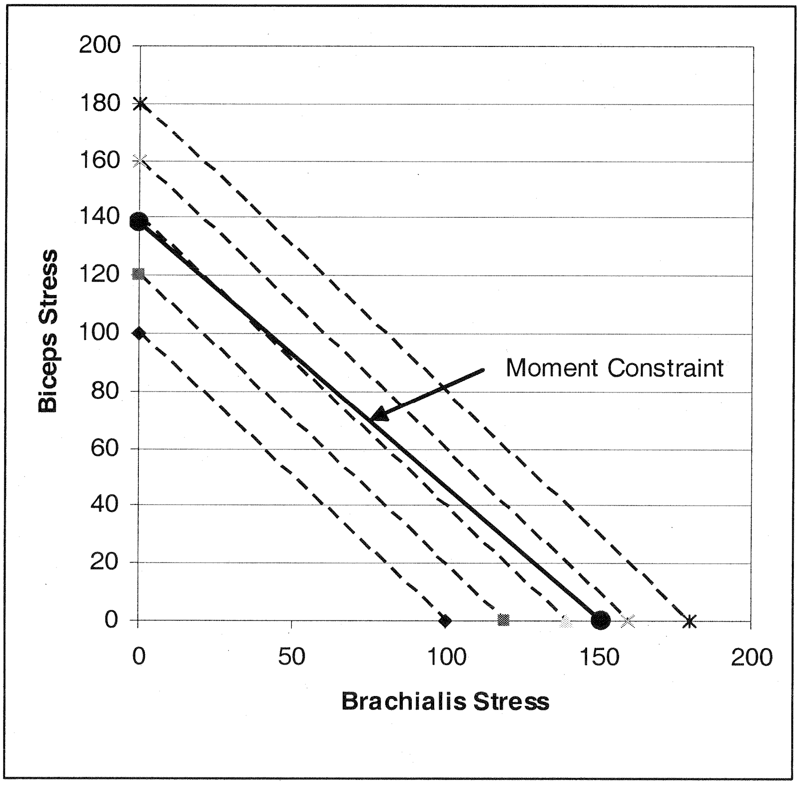
Minimize $S_{TOT} = S_{BR} + S_{BI} = F_{BR}/3.0 + F_{BI}/4.5$ N/cm^2

Subject to the moment equation:

$$\vec{r}_H \times \vec{W}_{H+B} + \vec{r}_{FA} \times \vec{W}_{FA} + \vec{r}_{BR} \times \vec{F}_{BR} + \vec{r}_{BI} \times \vec{F}_{BI} = \vec{0}$$
$$\vec{r}_H \times \vec{W}_{H+B} = (32\hat{i}) * (-5.5 * 9.81\hat{j}) = -1726\hat{k} \text{ N-cm}$$
$$\vec{r}_{FA} \times \vec{W}_{FA} = (13\hat{i}) * (-1.6 * 9.81\hat{j}) = -2.04\hat{k} \text{ N-cm}$$
$$\vec{r}_{BR} \times \vec{F}_{BR} = (5\hat{i} + 1\hat{j}) * F_{BR}[\cos(135)\hat{i} + \sin(135)\hat{j}] = 4.24 F_{BR}\hat{k} \text{ N-cm}$$
$$\vec{r}_{BI} \times \vec{F}_{BI} = (3\hat{i} + 1\hat{j}) * F_{BI}[\cos(120)\hat{i} + \sin(120)\hat{j}] = 3.10 F_{BR}\hat{k} \text{ N-cm}$$

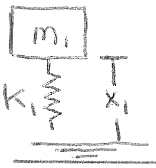
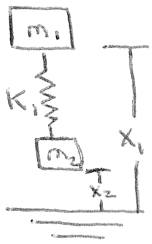
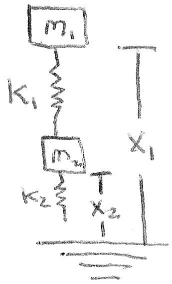
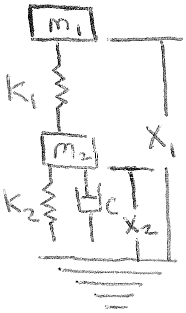
Plugging in the numerical values (similar to the homework problem) yields:

$4.24F_{BR} + 3.10F_{BI} = 1930$ N-cm, or in terms of stresses
$12.72S_{BR} + 13.95S_{BI} = 1930$ N-cm



Plotting values of the objective function $S_{TOT} = S_{BR} + S_{BI}$= constant, along with the moment equation demonstrates that the minimum stress will occur at the point where the Brachialis stress is zero and the Biceps stress is 138.5 N/cm^2, corresponding to a force of 622.6 N.

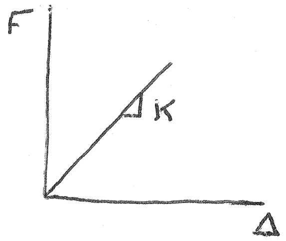
2.12

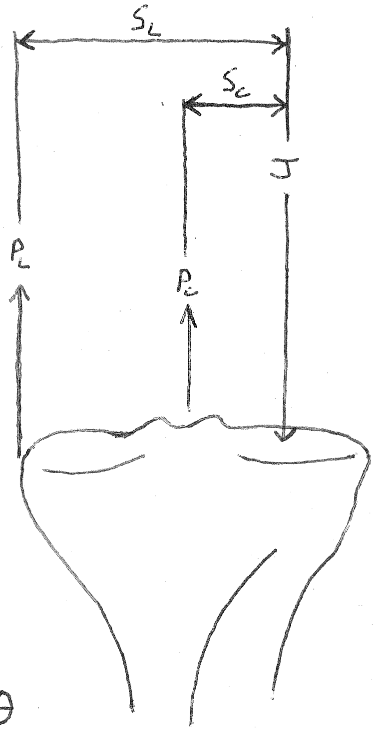
	Foot on ground	Foot in air
	$m_1\ddot{x}_1 + k_1x_1 + m_1g = 0$	$m_1\ddot{x}_1 + m_1g = 0$
	$m_1\ddot{x}_1 + k_1(x_1 - x_2) + m_1g = 0$ $m_2\ddot{x}_2 - k_1(x_1 - x_2) + m_2g = 0$	$m_1\ddot{x}_1 + k_1(x_1 - x_2) + m_1g = 0$ $m_2\ddot{x}_2 - k_1(x_1 - x_2) + m_2g = 0$
	$m_1\ddot{x}_1 + k_1(x_1 - x_2) + m_1g = 0$ $m_2\ddot{x}_2 - k_1(x_1 - x_2) + k_2x_2 + m_2g = 0$	$m_1\ddot{x}_1 + k_1(x_1 - x_2) + m_1g = 0$ $m_2\ddot{x}_2 - k_1(x_1 - x_2) + m_2g = 0$
	$m_1\ddot{x}_1 + k_1(x_1 - x_2) + m_1g = 0$ $m_2\ddot{x}_2 - k_1(x_1 - x_2) + k_2x_2 + C\dot{x} + m_2g = 0$	$m_1\ddot{x}_1 + k_1(x_1 - x_2) + m_1g = 0$ $m_2\ddot{x}_2 - k_1(x_1 - x_2) + m_2g = 0$

$x_1 = x_2 = 0$ when spring force equals 0

2.13

$$\frac{M_c}{M_L} = \frac{F_c S_c}{F_L S_L}$$





$\therefore$

$$\frac{M_c}{M_L} \approx \frac{K \Delta_c S_c}{K \Delta_L S_L} \qquad \Delta_c \approx S_c \theta \qquad \Delta_L \approx S_L \theta$$

$$\frac{M_c}{M_L} \approx \frac{K S_c^2 \theta}{K S_L^2 \theta} \approx \frac{S_c^2}{S_L^2}$$

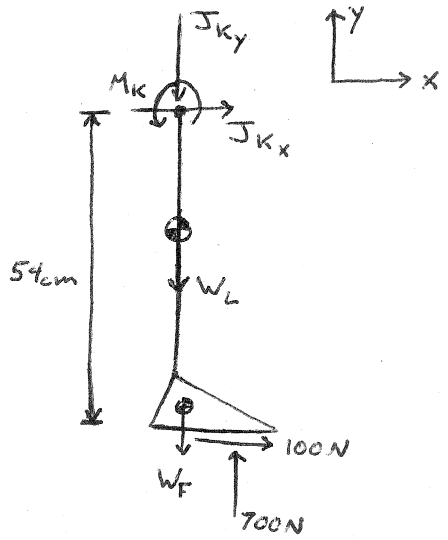
Approximate dimensions

$$\frac{M_c}{M_L} \approx \left(\frac{1}{3}\right)^2 \approx \frac{1}{9}$$

The lateral collateral ligament carries approximately nine times greater load than the cruciate ligaments.

2.14

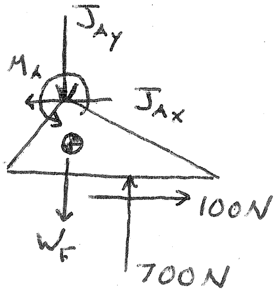
Quasi-static



$$\Sigma F_x: \; J_{Kx} + 100N = 0$$
$$J_{Kx} = -100N$$
$$\Sigma F_y: \; -J_{Ky} - 10N - 30N + 700N = 0$$
$$J_{Ky} = 660N$$
$$\Sigma M: \; M_K + 100N(0.54m) + 700N(0.08m) = 0$$
$$M_K = -110 \text{ N-m}$$

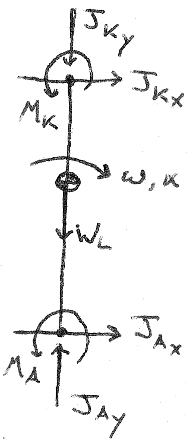
Dynamic

Foot is stationary in single-leg stance.



$$\Sigma F_x: \; -J_{Ax} + 100N = 0$$
$$J_{Ax} = 100N$$
$$\Sigma F_y: \; -J_{Ay} - 10N + 700N = 0$$
$$J_{Ay} = 690N$$
$$\Sigma M: \; M_A + 700N(0.08m) + 100N(0.10m) = 0$$
$$M_A = -66 \text{ N-m}$$

Lower leg is rotating about the ankle.



$$\Sigma F_x: \; 100N + J_{Kx} = \left(\frac{30N}{9.81}\right)(0.24m)(0.2 \text{ rad/s}^2)$$
$$J_{Kx} = -99.85N$$
$$\Sigma F_y: \; 690N - 30N - J_{Ky} = -\left(\frac{30N}{9.81}\right)(0.24m)(0.8 \text{ rad/s})^2$$
$$J_{Ky} = 660.47N$$
$$\Sigma M: \; 66 \text{ N-m} - J_{Kx}(0.44m) + M_K$$
$$= -\left(\frac{30N}{9.81}\right)\left[\frac{1}{12}(0.44m)^2 + (0.24m)^2\right](0.2 \text{ rad/s}^2)$$

2.14 cont'd

	Quasi-static	Dynamic	% Difference
J_{K_x}	-100 N	-99.85 N	0.15%
J_{K_y}	660 N	660.47 N	-0.071%
M_K	-110 N-m	-109.97 N-m	0.027%

The difference between quasi-static and dynamic analysis is negligible.